# The Twenty-Ninth Annual
# Eastern Shore High School Mathematics Competition

November 8, 2012

## Individual Contest Exam

Instructions

There are twenty problems on this exam. Select the best answer for each problem.

Your score will be the number of *correct* answers that you select.
**There is no penalty for incorrect answers.**

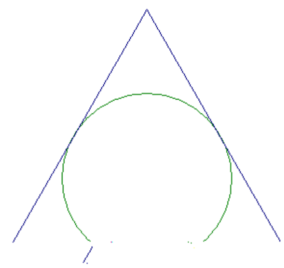
The use of a calculator is **not** permitted on this exam.

In the event of tie scores, #18, #19 and #20 will be used as tiebreakers.

1. Suppose $f \quad g(x) = x^2 \quad x + 1$ and $f(x) = x^2 + x + 1$. $g(x)$ is

(a) $\quad x$ (b) $x \quad 1$ (c) $1 \quad x$ (d) Both (a) and (b) (e) Both (a) and (c)

2. The image below shows an equilateral triangle with an inscribed circle.



Suppose the height of the triangle is 18. The radius of the inscribed circle is

(a) $3\sqrt{3}$ (b) 6 (c) $4\sqrt{3}$ (d) $3\sqrt{6}$ (e) 8

3. Suppose $\log_2 x + 2\log_4 x + 3\log_8 x + 4\log_{16} x + 5\log_{32} x = 20$. Solve for x.

(a) 2 (b) 4 (c) 8 (d) 16 (e) 32

4. Let $a$ and $b$ be nonzero real numbers such that $a > b$. Which of the following *must* be true?

(a) $(a + 2)^3 > (b + 2)^3$ (b) $\sqrt{|a|} > \sqrt{|b|}$ (c) $\sin(a) > \sin(b)$ (d) $\frac{a}{b} > 1$ (e) $a^2 > b^2$

5. Suppose a whole number $n$ is quadrupled. Then, $\frac{1}{\sqrt{n}}$ is

(a) halved (b) doubled (c) quartered (d) quadrupled (e) None of these

6. The graph of $y = \sec x$ will coincide with the graph of $y = \sec(x \quad \frac{1}{2}\pi)$ if the graph of $y = \sec x$ is shifted to the

(a) left $\frac{1}{2}\pi$ units (b) right $\frac{1}{2}\pi$ units (c) left $2\pi$ units (d) right $2\pi$ units (e) left $\pi$ units

7. In the decimal expansion of $3^{2012} \quad 2^{2012}$, what digit is in the units place?

(a) 1 (b) 3 (c) 5 (d) 7 (e) 9

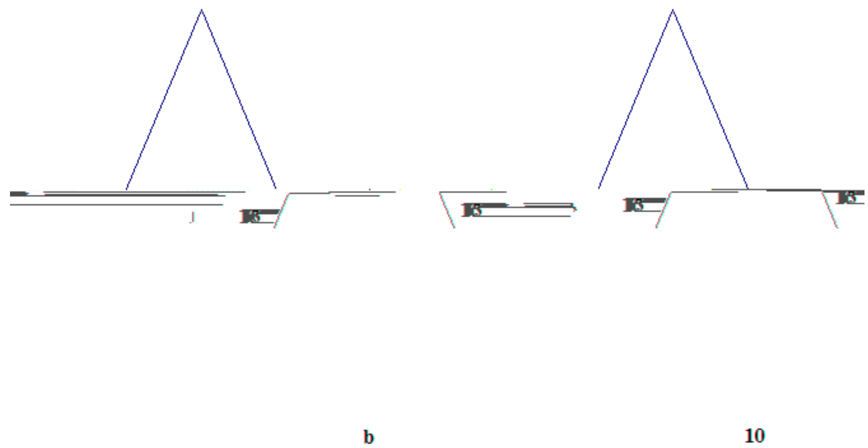
8. A triangle with vertices (10, 4), (14, 4) and (14, 8) is reflected over the line $y = x$. The resulting image is then reflected over the $y$-axis. The coordinates of the final image of the vertex at (14, 4) are

(a) (-4, 14) (b) (-14, -4) (c) (-4, -14) (d) (4, -14) (e) (14, 4)

9. Suppose $a$ and $c$ are given positive integers. For which of the following values of $b$ can $ax^2 + bx + c$ always be factored into linear factors using **only integers**?

(a) $b = \sqrt{2ac}$ (b) $b = \sqrt{4ac}$ (c) $b = 0$ (d) $b = ac + 1$ (e) $b = -2a$

10. What choice of $b$ will result in the two triangles below having the same area?
Note: figures may not be drawn to scale.




(a) 5 (b) 6 (c) 12 (d) 24 (e) 30

11. How many real solutions $(x, y)$ does the following pair of equations have?

$$y = (x - 5)^2 + 3$$
$$y = -(x - 5)^2 + 17$$

(a) 0 (b) 1 (c) 2 (d) 3 (e) None of these

12. Suppose all Gigglers boggle. Which of the following must be true?

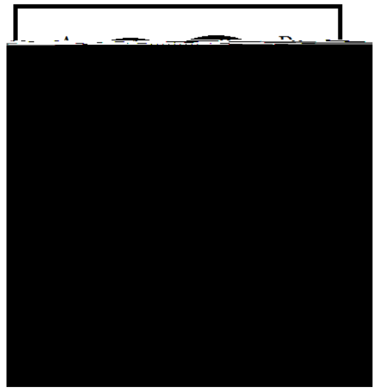
I. If it is a Giggler, it boggles.
II. If it boggles, it is a Giggler.
III. All those who boggle are Gigglers.
IV. No one who does not boggle is a Giggler.
V. If it doesn't boggle, it is not a Giggler.

(a) All all of these must be true
(b) Only I
(c) Only III
(d) Only I, IV, and V
(e) Only II and III

13. The digits 1 through 5 are shuffled and a five digit number is written down. Each digit is used exactly once. What is the probability that the number is divisible by 2 or 5?

(a) $\frac{1}{120}$ (b) $\frac{2}{5}$ (c) $\frac{1}{2}$ (d) $\frac{3}{5}$ (e) None of these

14. In the figure below, the shaded region represents which of the following?



(a) $A \cap B \cap C'$ (b) $B \cap$

19. Consider a triangle with one side of length 16 and one side of length 20 such that the measure of an angle opposite one of these two sides is 30 degrees. How many triangles satisfy these conditions?

(a) 0 (b) 1 (c) 2 (d) 3 (e) Cannot be determined

20. Suppose $a$ is the sixth term of a non-constant geometric sequence and $b$ is the ninth term. The rst term of the sequence is

(a) $\frac{a^8}{b^5}{}^{1=3}$ (b) $\frac{8a \quad 5b}{3}$ (c) $\frac{4a \quad 5b}{3}$ (d) $\frac{1}{3} \; \frac{a^8}{b^5}$ (e) $\frac{b}{a}{}^{1=3}$